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# Selective Disclosure for Identity Management

# A critique of identity

- Identity as a proxy to check credentials
  - Username decides access in Access Control Matrix
- Sometimes this leaks too much information
- Real world examples
  - Tickets allow you to use cinema / train
  - Bars require customers to be older than 18
    - But do you want the barman to know your address?

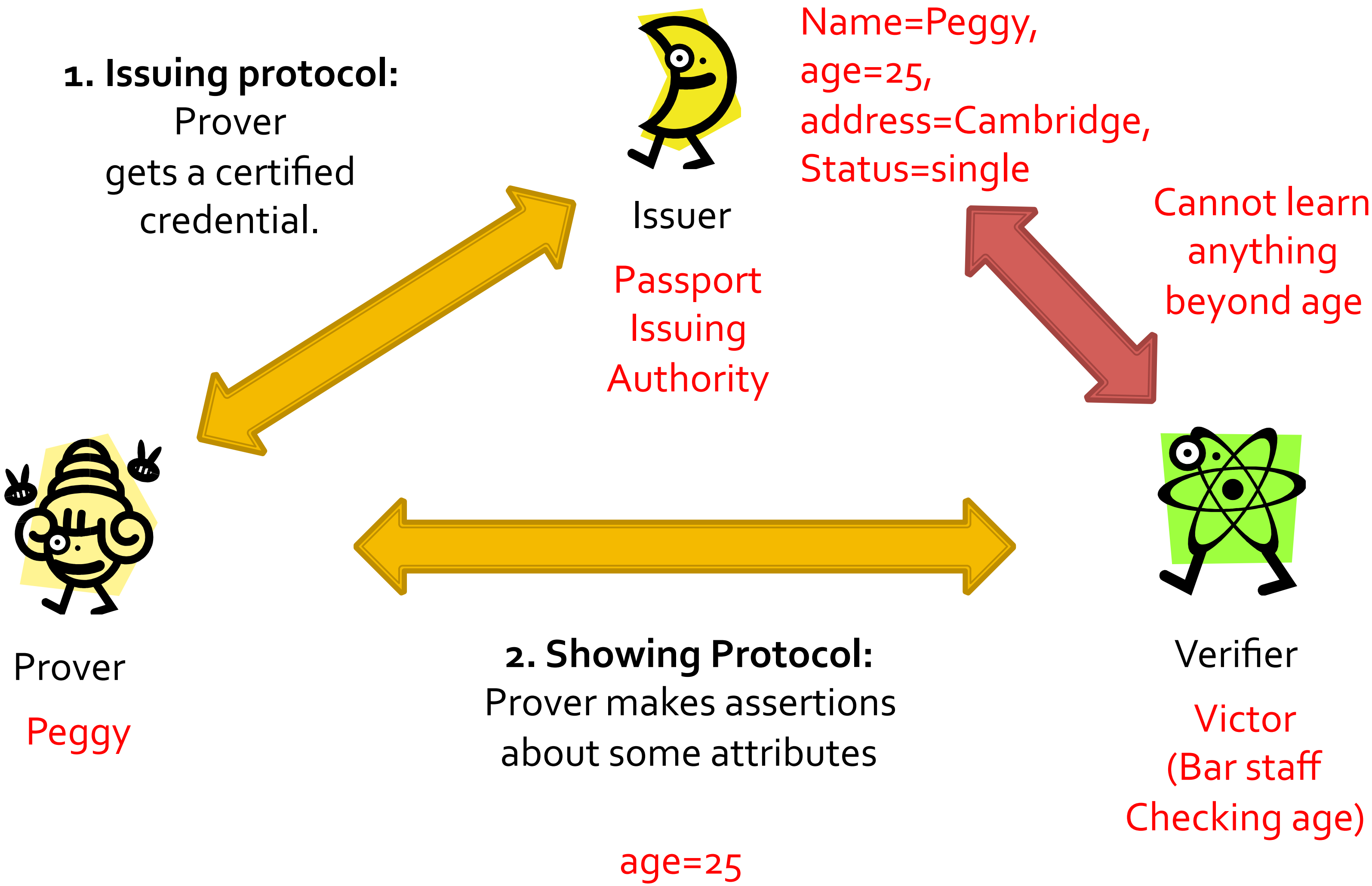
# The privacy-invasive way

- Usual way:
  - **Identity provider** certifies attributes of a **subject**.
  - **Relying Party** checks those attributes
  - Match credential with **live person** (biometric)
- Examples:
  - E-passport: signed attributes, with lightweight access control.
    - Attributes: nationality, names, number, pictures, …
  - Identity Cards: signatures over attributes
    - Attributes: names, date of birth, picture, address, …

# Selective Disclosure Credentials

- The players:
  - Issuer (I) = Identity provider
  - Prover (P) = Subject
  - Verifier (V) = Relying party
- Properties:
  - The prover convinces the verifier that he holds a credential with attributes that satisfy some boolean formula:
    - Simple example "age=18 AND city=Cambridge"
  - Prover cannot lie
  - Verifier cannot infer anything else aside the formula
  - Anonymity maintained despite collusion of V & I

# The big picture



# Two flavours of credentials

- Single-show credential (Brands & Chaum)
  - Blind the issuing protocol
  - Show the credential in clear
  - Multiple shows are linkable – BAD
- Multi-show (Camenisch & Lysyanskaya)
  - Random oracle free signatures for issuing (CL)
  - Blinded showing
    - Prover shows that they know a signature over a particular ciphertext.
  - Cannot link multiple shows of the credential
  - More complex – BAD

We will Focus on these

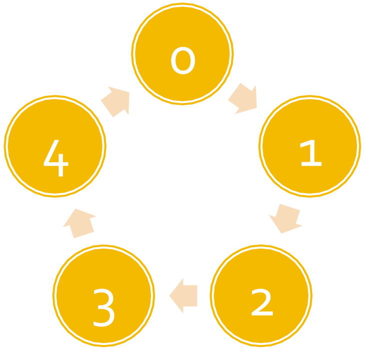
# Technical Outline

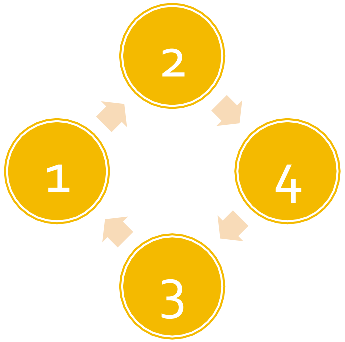
- Cryptographic preliminaries
  - The discrete logarithm problem
  - Schnorr's Identification protocol
    - Unforgeability, simulator, Fiat-Shamir Heuristic
    - Generalization to representation
- Showing protocol
  - Linear relations of attributes
  - AND-connective
- Issuing protocol
  - Unlikable issuing
  - Efficient proof of a signature.

What is a Zero-Knowledge Proof?

# Discrete logarithms (I) - revision

- Assume $p$ a large prime
  - (>1024 bits—2048 bits)
  - Detail: $p = qr+1$ where $q$ also large prime
  - Denote the field of integers modulo $p$ as $Z_p$
- Example with $p$=5
  - Addition works fine: $1+2 = 3$, $3+3 = 1$, ...
  - Multiplication too: $2*2 = 4$, $2*3 = 1$, ...
  - Exponentiation is as expected: $2^2 = 4$
- Choose $g$ in the multiplicative group of $Z_p$
  - Such that $g$ is a generator
  - Example: g=2



# Discrete logarithms (II) -revision

- Exponentiation is computationally easy:
  - Given $g$ and $x$, easy to compute $g^x$
- But logarithm is computationally hard:
  - Given $g$ and $g^x$, difficult to find $x = \log_g g^x$
  - If $p$ is large it is practically impossible
- Related DH problem
  - Given $(g, g^x, g^y)$ difficult to find $g^{xy}$
  - Stronger assumption than DL problem

# More on $Z_p$

- Efficient to find inverses
  - Given c easy to calculate $g^{-c}$ mod p
    - (p-1) – c mod p-1
- Efficient to find roots
  - Given c easy to find $g^{1/c}$ mod p
    - c (1/c) = 1 mod (p-1)
  - Note the case N=pq (RSA security)
- No need to be scared of this field.

# Schnorr's Identification protocol

- Exemplary of the zero-knowledge protocols credentials are based on.
- Players
  - Public – g a generator of $Z_p$
  - Prover – knows x (secret key)
  - Verifier – knows $y = g^x$ (public key)
- Aim: the prover convinces the verifier that she knows an x such that $g^x = y$
  - Zero-knowledge – verifier does not learn x!
- Why identification?
  - Given a certificate containing y

# Schnorr's protocol

Public: g, p

Knows: x

Peggy
(Prover)

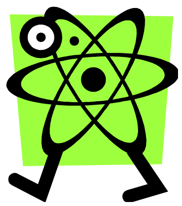
Random: w

P->V: $g^w = a$ (witness)

V->P: c (challenge)

P->V: $cx+w = r$ (response)

Knows: $y=g^x$



Victor
(Verifier)

Check:

$g^r = y^c a$

↓

$g^{cx+w} = (g^x)^c g^w$

# No Schnorr Forgery (intuition)

- Assume that Peggy (Prover) does not know x?
  - If, for the same witness, Peggy forges two valid responses to two of Victor's challenges

$$r_1 = c_1 x + w$$

$$r_2 = c_2 x + w$$

  - Then Peggy must know x
    - 2 equations, 2 unknowns (x,w) – can find x

# Zero-knowledge (intuition)

- The verifier learns nothing new about x.
- How do we go about proving this?
  - Verifier can simulate protocol executions
    - On his own!
    - Without any help from Peggy (Prover)
  - This means that the transcript gives no information about x
- How does Victor simulate a transcript?
  - (Witness, challenge, response)

# Simulator

- Need to fake a transcript ($g^{w'}$, c', r')
- Simulator:
  - Trick: do not follow the protocol order!
  - First pick the challenge c'
  - Then pick a random response r'
    - Then note that the response must satisfy:

      $g^{r'} = (g^x)^{c'} g^{w'} \rightarrow g^{w'} = g^{r'} / (g^x)^{c'}$
  - Solve for $g^{w'}$
- Proof technique for ZK
  - but also important in constructions (OR)

# Non-interactive proof?

- Schnorr's protocol
  - Requires interaction between Peggy and Victor
  - Victor cannot transfer proof to convince Charlie
    - (In fact we saw he can completely fake a transcript)
- **Fiat-Shamir Heuristic**
  - $H[\cdot]$ is a cryptographic hash function
  - Peggy sets $c = H[g^w]$
  - Note that the simulator cannot work any more
    - $g^w$ has to be set first to derive c
- Signature scheme
  - Peggy sets $c = H[g^w, M]$

# Generalise to DL represenations

- Traditional Schnorr
  - For fixed g, p and public key $h = g^x$
  - Peggy proves she knows x such that $h = g^x$
- General problem
  - Fix prime p, generators $g_1, \ldots, g_l$
  - Public key $h' = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$
  - Peggy proves she knows $x_1, \ldots, x_l$ such that $h' = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$

# DL represenation – protocol

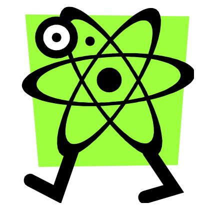
Public: g, p

Knows: $x_1, \ldots, x_l$

Peggy
(Prover)

l random: $w_i$

Knows:
$h = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$



Victor
(Verifier)

P->V: $\prod_{0<i<l} g^{w_i} = a$ (witness)

V->P: c (challenge)

$r_i = cx_i + w_i$

P->V: $r_1, \ldots, r_l$ (response)

Check:

$$\left(\prod_{0<i<l} g_i^{r_i}\right) = h^c a$$

Let's convince ourselves: $(\prod_{0<i<l} g_i^{r_i}) = (\prod_{0<i<l} g_i^{x_i})^c (\prod_{0<i<l} g^{w_i}) = h^c a$

# DL represenation vs. Schnorr

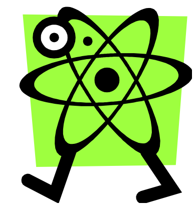
Public: g, p

Knows: $x_1, \ldots, x_l$

Peggy
(Prover)

l random: $w_i$

Knows:
$h = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$



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# Credentials – showing

- Relation to DL representation
- Credential representation:
  - Attributes $x_i$
  - Credential $h = g_1^{x_1} g_2^{x_2} \dots g_l^{x_l}$, $Sig_{Issuer}(h)$
- Credential showing protocol
  - Peggy gives the credential to Victor $(h, Sig_{Issuer}(h))$
  - Discloses only some attributes
  - Peggy proves a statement on values $x_i$
    - $X_{age} = 28$ AND $x_{city} = H[Cambridge]$

# How?

- It always reduces to proving knowledge of a DL representation.
  - But which one?
- To simply disclose attributes
  - Cancel them out of the credential
  - For $X_{age} = 28$ AND $x_{city} = H[Cambridge]$
- Proves she know the DL representation of

$$h/(g_{age})^{x_{age}}(g_{city})^{x_{city}} = h' = \prod_{3<i<l} g^{x_i}$$

(Also do not forget to check the signature!)

# Linear relations of attributes (1)

- Remember:
  - Attributes $x_i$ , i = 1,…,4
  - Credential $h = g_1^{x_1} g_2^{x_2} g_3^{x_3} g_4^{x_4}$, $Sig_{Issuer}(h)$
- Example relation of attributes:
  - $(x_1 + 2x_2 - 10x_3 = 13)$ AND $(x_2 - 4x_3 = 5)$
  - Implies: $(x_1 = 2x_3+3)$ AND $(x_2 = 4x_3+5)$
  - Substitute into h
    - $h = g_1^{2x_3+3} g_2^{4x_3+5} g_3^{x_3} g_4^{x_4} = (g_1^3 g_2^5)(g_1^2 g_2^4 g_3)^{x_3} g_4^{x_4}$
    - Implies: $h / (g_1^3 g_2^5) = (g_1^2 g_2^4 g_3)^{x_3} g_4^{x_4}$

# Linear relations of attributes (2)

- Example (continued)
  - $(x_1 + 2x_2 - 10x_3 = 13)$ AND $(x_2 - 4x_3 = 5)$
  - Implies: $h / (g_1^3 g_2^5) = (g_1^2 g_2^4 g_3)^{x_3} g_4^{x_4}$
- How do we prove that in ZK?
  - DL representation proof!
    - $h' = h / (g_1^3 g_2^5)$
    - $g_1' = g_1^2 g_2^4 g_3$ $\qquad g_2' = g_4$
  - Prove that you know $x_3$ and $x_4$ such that $h' = (g_1')^{x_3} (g_2')^{x_4}$

# DL rep. – credential show example

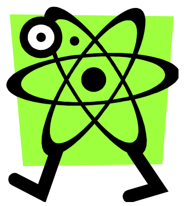
Public: g, p

Knows: $x_1, x_2, x_3, x_4$

Peggy
(Prover)

random: $w_1, w_2$

Knows:
$h = g_1^{x_1} g_2^{x_2} g_3^{x_3} g_4^{x_4}$



Victor
(Verifier)

P->V: $g_1'^{w_1} g_2'^{w_2} = a'$ (witness)

V->P: c (challenge)

$r_1 = cx_3 + w_1$

$r_2 = cx_4 + w_2$

P->V: $r_1, r_2$ (response)

Check:

$$(g_1')^{r_1} (g_2')^{r_2} = (h')^c a$$

# Check $(g_1')^{r_1} (g_2')^{r_2} = (h')^c a$

- Reminder
  - $h = g_1^{X_1} g_2^{X_2} g_3^{X_3} g_4^{X_4}$
  - $h' = h / (g_1^3 g_2^5)$ $\quad g_1' = g_1^2 g_2^4 g_3$ $\quad g_2' = g_4$
  - $a = g_1'^{w_1} g_2'^{w_2}$ $\quad r_1 = cx_3 + w_1$ $\quad r_2 = cx_4 + w_1$
- Check:
  - $(g_1')^{r_1} (g_2')^{r_2} = (h')^c a \Rightarrow$
    $(g_1')^{(\cancel{c}x_3+\cancel{w}_1)} (g_2')^{(\cancel{c}x_4+\cancel{w}_1)} = (h / (g_1^3 g_2^5))^{\cancel{c}} \cancel{g_1}'^{w_1} \cancel{g_2}'^{w_2} \Rightarrow$
    $(g_1^{2x_3+3} g_2^{4x_3+5} g_3^{x_3} g_4^{x_4}) = h$

$\downarrow$ $x_1$ $\qquad$ $\downarrow$ $x_2$

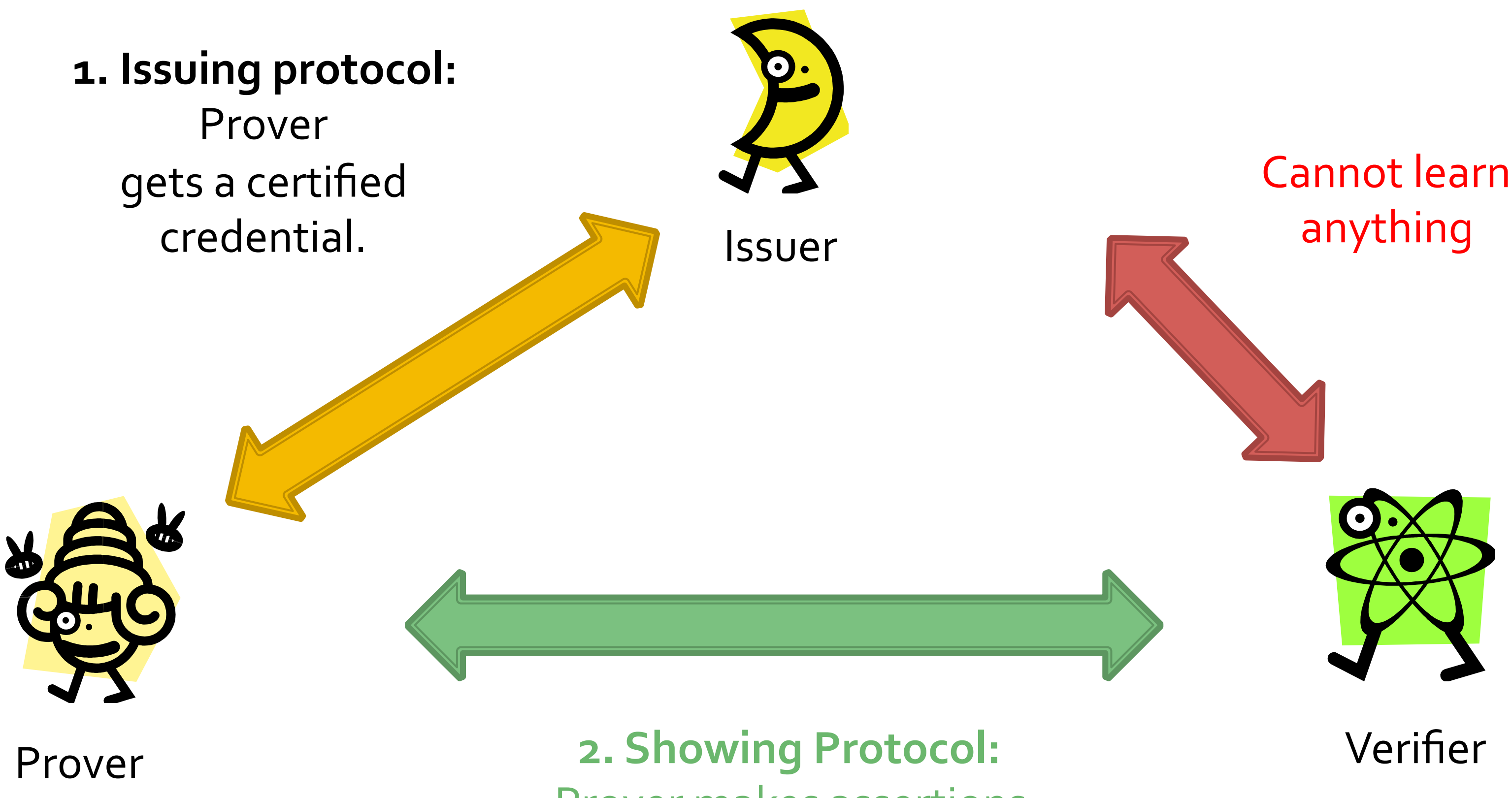
# A few notes

- Showing any relation implies knowing all attributes.
- Can make non-interactive (message m)
  - c = H[h, m, a']
- Other proofs:
  - (OR) connector *(simple concept)*
    - $(x_{age}=18 \text{ AND } x_{city}=H[\text{Cambridge}]) \text{ OR } (x_{age}=15)$
  - (NOT) connector
  - Inequality $(x_{age} > 18)$

# Summary of key concepts (1)

- Standard tools
  - Schnorr – ZK proof of knowledge of discrete log.
  - DL rep. – ZK proof of knowledge of representation.
- Credential showing
  - representation + certificate
  - ZK proof of linear relations on attributes (AND)
  - More reading: (OR), (NOT), Inequality

# Issuing credentials




Credential

$h = g_1^{X_1} g_2^{X_2} \ldots g_l^{X_l}$

$Sig_{Issuer}(h)$

# Issuing security

- Issuing: What do we want?
  - Peggy authenticates and provides a list of attributes.
  - Issue checks all and provides a signed credential.
    - In the form we discussed previously.
- Peggy needs to do two things:
  - Blind the credential.
    - Multiple times
  - Prove that she possesses a valid signature on it.
    - Without revealing the actual signature.
- Solution: the CL signature scheme.

# CL Signature Scheme

- Setup:
  - Generate and RSA modulus $n = pq$
    (with $p=2p'+1$, $q=2q'+1$, $p,q,p',q'$ large primes)
  - Choose $g_1, \ldots, g_l, b, c$
    (all of which are quadratic residues)
  - Public key = $(n, g_1, \ldots, g_l, b, c)$;
    Private Key = $p, q$

- Signature:
  - Attributes: $x_1, \ldots, x_l$
  - Pick a random prime e, and random s
  - $v = (c / ((g_1)^{x_1} \ldots (g_l)^{x_l}\, b^s)^{1/e} \bmod n$
  - Output signature $(e, s, v)$
    - Cannot forge because $(.)^{1/e}$ requires knowledge of p, q

# How to verify a CL signature?

- Reminder
  - Public: c, $g_i$, b, n
  - $v = (c / ((g_1)^{x_1} \ldots (g_l)^{x_l} b^s)^{1/e} \bmod n$
  - Signature (e, s, v)
- Zero-knowledge DL Rep. Proof:
  - Get a random r
  - Define $v' = v\, b^r$
  - Reveal: v'
  - DL Rep. proof of:
    $c = (v')^e ((g_1)^{x_1} \ldots (g_l)^{x_l} b^{s-er}$

# Does that work?

- $c = (v')^e ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^{s-er}$
  - $c = (v\, b^r)^e ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s\, b^{-er}$
  - $c = (v)^e (b^{re}) ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s\, b^{-er}$
    - Remember: $v = (c / ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s )^{1/e}$
  - $c = ((c / ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s )^{1/e} )^e ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s$
  - $c = (c / ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s ) ((g_1)^{x_1} \dots (g_l)^{x_l}\, b^s$
  - $c = c$

# Unforgeability of signature

Based on Strong RSA assumption:

- Impossible to find a v'
- Without computing $(.)^{1/e}$
- Which is infeasible without p, q
- Prover does not know p, q (only n)

# Privacy

- Unlikability of signature and showing
  - Signature (e,s,v)
  - Showing (v’) + ZK proof
    - V and v’ are unlinkable
    - Proof does not learn s, e
- Result:
  - We can show the credential many times.
  - Each time is unlikable to the others.
  - One issue – many (unlinkable) uses.

# Full credential protocol

- Putting it all together:
  - CL signature proof is already a DL proof:

    $$c = (v')^e ((g_1)^{x_1} \ldots (g_l)^{x_l}\, b^{s-er}$$

  - Integrate all previous tricks to reveal or show relations on attributes.
  - E.g. show attributes $x_1$ and $x_2$:
    - Reveal $x_1$ and $x_2$
    - Show $c \,/\, (g_1)^{x_1}(g_2)^{x_2} = (v')^e ((g_3)^{x_3} \ldots (g_l)^{x_l}\, b^{s-er}$

# Key concepts so far (2)

- Credential issuing
  - Authentication & Authoritzation
  - Signing (using CL)
- Showing Credential
  - Re-randomize and proof possession of signature
  - Integrate proof over attributes
- Further topics
  - Transferability of credential
  - Double spending

# Key applications

- Attribute based access control
- Federated identity management
- Electronic cash
  - (double spending)
- Privacy friendly e-identity
  - Id-cards & e-passports
- Multi-show credentials!

# References

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